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### Analytical modelling and laboratory studies of particle transport in filter media

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# **Analytical Modelling and Laboratory Studies of Particle Transport in Filter Media**

A thesis submitted in fulfilment of the  
requirement for the award of the degree

**Doctor of Philosophy**



from

**UNIVERSITY OF WOLLONGONG**



by

**Fereydoon Vafai, B.Sc., M.Sc.**

Department of Civil and Mining Engineering

1996

# IN THE NAME OF GOD



**AFFIRMATION**

I hereby certify that the work presented in this thesis is original and has been carried out in the Department of Civil and Mining Engineering of the University of Wollongong and has not been submitted for any other degree.

.....

Fereydoon Vafai

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## **PUBLICATIONS THROUGH THIS STUDY**

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Indraratna, B. and Vafai, F. (1996). Analytical model for predicting particle migration within a base soil-filter system. J. of Geotechnical Engineering. American Society of Civil Engineers, (in press).

Indraratna, B., Vafai, F. and Haque, A. (1996). Design of granular filters based on experimental and analytical studies. GEOFILTERS'96: 2nd Int. Conference. Montreal, Canada, pp 1-10.

Indraratna, B., Vafai, F. and Haque, A. (1996). Laboratory and Analytical modelling of granular filters. 8th ANZ Conference in Geomechanics, Adelaide, Australia (June, 1996).

## ABSTRACT

This study highlights an analytical model simulating the filtration phenomenon applicable to any base soil-filter system. Prior to development of such a model, different approaches in filter design criteria are reviewed, and their advantages and disadvantages are critically discussed. The mechanics of filtration are investigated both analytically and experimentally to achieve a better understanding of the behaviour of particle migration within the filter medium, and highlight the influencing parameters affecting the filtration process. The evaluation of pore size and coefficient of permeability of the filter are studied, and a new relationship is established to determine the coefficient of permeability based on finer fraction of particle size distribution curve ( $D_5$  and  $D_{10}$ ). Considering the theoretical concepts of filtration phenomenon, the proposed model includes the actual hydraulic conditions and the relevant material properties such as: coefficient of permeability, porosity, friction angle, and the shape and distribution of particles. The model is founded on the concept of critical hydraulic gradient derived from limit equilibrium considerations, where the migration of particles is assumed to occur under applied hydraulic gradients exceeding this critical value. The rate of particle erosion, and hence, the filter effectiveness is quantified on the basis of mass and momentum conservation theories. By dividing the base soil and the filter domains into discrete elements, the model is capable of predicting the time-dependent particle gradation and permeability of each element, thereby the amount of material eroded from or retained within a given soil-filter system. Laboratory tests conducted on a fine base material verified the validity of the model. A design procedure for efficient filter using the



proposed model is presented in a separate chapter for two dimensional flow in a simplified earth structure. The model predictions are also compared with the commonly used empirical recommendations, including the conventional grading ratios. The response of the model in relation to the self-filtration phenomenon is also discussed. Finally, recommendations for further research are given in the context of the findings of this study.

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## LIST OF SYMBOLS AND ABBREVIATIONS

$D_h$	equivalent effective diameter
$D_i$	average diameter in the i-th interval in particle size distribution curve
$N$	number of particles;
$G_i$	specific gravity of i-th particle;
$\gamma_s$	unit weight of soil.
$\alpha$	shape coefficient of particle
$d_a$	average pore diameter or average diameter of pore channel
$d_{min}$	minimum diameter of the pore channel
$d_{max}$	maximum diameter of the pore channel
$V_p$	pore volume
$V$	total volume of sample
$n$	porosity
$\Delta l$	length of of the sample in the flow direction
$N'$	number of channels
$\Delta L'$	average length of the pore channels
$T$	parameter of tortuosity
$k$	coefficient of permeability (cm/sec or ft/day)
$K$	intrinsic permeability factor (cm <sup>2</sup> or ft <sup>2</sup> )

$\mu$  viscosity of permeant

$e$  void ratio

$k_T, k_{20}$  coefficient of permeability at  $T^\circ\text{C}$  and  $20^\circ\text{C}$  respectively

$\eta_T, \eta_{20}$  viscosity of liquid at  $T^\circ\text{C}$  and  $20^\circ\text{C}$  respectively

$\gamma$  unit weight of permeant

$S$  specific surface =  $\frac{6}{\sqrt{d_1/d_2}}$  .  $d_1, d_2$  = maximum and minimum sized particles

$f$  angularity factor for rounded grains

$k_{0.85}$  coefficient of permeability at a void ratio of 0.85.

$r^2$  regression coefficient, decreases).

$Q$  discharge capacity

$\Delta h$  head loss through each layer,

$H$  total head loss

$R$  hydraulic radius

$C$  correction factor for coefficient of permeability in turbulent flow

$k'$  effective coefficient of permeability

$i$  actual hydraulic gradient.

$\gamma_s h_s - \gamma_w h_w$  effective vertical stress

$h_s$  height of soil layer above the element

$\gamma_w$  unit weight of water

$h_w$  height of water above the element

$\sigma_x, \sigma_y$  lateral stress

$\sigma_z$  vertical stress

$\lambda$  constant factor

$\phi'$  effective friction angle of the material

$W$  weight of the particle

$F_u$  uplift force

$\Delta P$  net hydrodynamic force

$\rho_m$  density of slurry

$\rho_w, \rho_s$  density of water and soil grains, respectively

$V_w$  volume of water in each element, and

$V_s$  summation of the volume soil particles of which the diameter is smaller than  $d_{min}$ .

$A$  cross section of the element

$u$  average velocity of slurry entering element.

$\Delta t$  time interval

$\Sigma F$  summation of external forces,

$V_m$  volume of the slurry in each element,

$dz$  width of element in the flow direction.

$g$  gravity acceleration



R	viscous drag per unit mass of slurry
Q	effluent flow rate
$V_{out}$	volume of effluent
$V_{s_{out}}$	volume of soil eroded from element
$V_{w_{out}}$	volume of water coming out of element
$P_j$	net volume of soil corresponding to a specific diameter j divided by the total volume of soil in that element ( $V_s$ ), and Cumulative value of $P_j$ is unity.
PSD	particle size distribution of the soil
$V_w$	volume of water in element
$V_T$	total volume of element
$D_{50}$	particle size in filter for which 50% by weight of particles are smaller
$d_{50}$	particle size in base for which 50% by weight of particles are smaller
$D_{15}$	particle size in filter for which 15% by weight of particles are smaller
$d_{15}$	particle size in base for which 15% by weight of particles are smaller
$d_{85}$	particle size in base for which 85% by weight of particles are smaller
$k_x$	coefficient of permeability in X direction
$k_z$	coefficient of permeability in Z direction
h	water pressure head
$\Delta x$	length of element in X direction
$\Delta z$	length of element in Z direction

$k_e$	equivalent coefficient of permeability of the system,
$\Delta l_m$	length of $m^{\text{th}}$ element,
$k_m$	coefficient of permeability of $m^{\text{th}}$ element, and
$L$	$\Sigma \Delta l$ .
$F_y$	force resulting due to the lateral stress, $\sigma_y$
$F_g$	effective weight
$F_f$	friction force between the particle and flow channel boundary
$\Delta P$	net hydrodynamic force
$i_\alpha$	mean hydraulic gradient acting within element in $\alpha$ direction
$i_x$	mean hydraulic gradient for horizontal seepage
$(i_x)_{cr}$	critical hydraulic gradient for horizontal seepage
$R'$	hydrodynamic number
$\nu$	kinematics viscosity coefficient